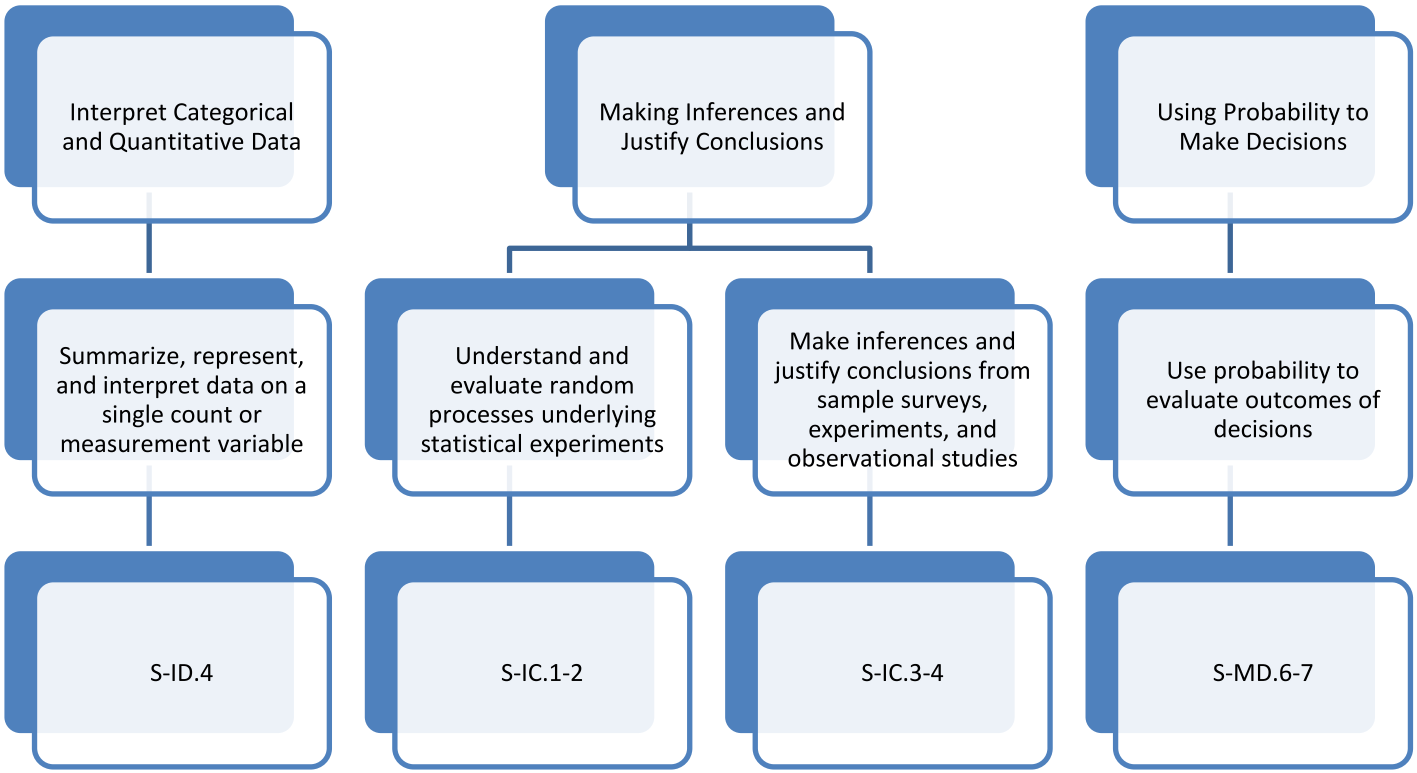
Honors Advanced Mathematics
Unit 3
Statistics and Probability



Honors Advanced Mathematics – UNIT 3
Statistics and Probability

Critical Area:

Students analyze data to make sound statistical decisions based on probability models. By investigating examples of simulations of experiments and observing outcomes of the data, students gain an understanding of what it means for a model to fit a particular data set. Students develop a statistical question in the form of a hypothesis (supposition) about a population parameter, choose a probability model for collecting data relevant to that parameter, collect data, and compare the results seen in the data with what is expected under the hypothesis. Students build on their understanding of data distributions to help see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). In addition, they can learn through examples the empirical rule, that for a normally distributed data set, 68% of the data lies within one standard deviation of the mean, and that 95% are within two standard deviations of the mean.

CLUSTERS	COMMON CORE STATE STANDARDS
(s)Summarize, represent, and interpret data on a single count or measurement data.	**Statistics and Probability – Interpreting Categorical and Quantitative Data** **S.ID.4.** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
Understand and evaluate random processes underlying statistical experiments. **Make inferences and justify conclusions from sample surveys experiments, and observational studies.**	**Statistics and Probability – Making Inferences and Justifying Conclusions** **S.IC.1.** Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. **S.IC.2.** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **S.IC.3.** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S.IC.4.** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S.IC.5.** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **S.IC.6.** Evaluate reports based on data**.**
Use probability to evaluate outcomes of decisions.	**Statistics and Probability – Using Probability to Make Decisions** **S.MD.6.** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S.MD.7.** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize MP 1, 2, 3, 4, 5, 6, and 7 in this unit.

LEARNING PROGRESSIONS
Draft High School Progression on Statistics and Probability http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• In real life, data sets are large and almost always approximately normal. Normal models which include estimation of areas under the normal curve allow us to answer and model real life situations. • Sampling methods, when highly representative of a population, allow accurate predictions or inferences of population parameters. • Students model probabilities found in experimental environment and decide whether they are consistent with theoretical probabilities? • The mean or proportion of a sample is the same as the mean or proportion of a population, within a margin of error. • If the difference between the statistics of two	• Why do we study normal distributions? • Why is random sampling of a population done when a census is impractical? • Do experimental probabilities match theoretical probabilities? • How can a researcher select a method of collecting data with as little bias as possible? • How does the mean or proportion of a sample compare to the mean or proportion of the population? • When does a statistic become extraordinary instead of ordinary? • How do you know when the difference between two treatments is statistically significant.	Bell curve bias categorical data census complementary events conditional probability confidence interval convenience sample correlation coefficient counting methods critical value of z distribution experimental probability experimental study

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
treatments is outside of a critical confidence interval, the difference is statistically significant. • Select a method of gathering data from a random sample and understand data by critically differentiating the merit of reports and data encountered in daily life. • Probability can be used to develop strategies and make informed decisions.	• There are many "studies out there", how do I know if they are really accurate? • How can probability be used to make fair decisions?	fairness Histogram independence independent events margin of error mean (x-bar) normal model or normal distribution null hypothesis Numerical data observational study parameter population probability distribution proportion (p-hat) qualitative data random number generator random sample random variable representative sample sampling significant (as in statistics) simple random sample standard deviation statistic stratified random sample Subject survey systematic random sample theoretical probability treatment voluntary sample Z-Score

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: **California Revised Mathematics Framework:** http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. **Illustrative Mathematics:** School Advisory Panel: S-IC.1 http://www.illustrativemathematics.org/illustrations/186 Strict Parents: S-IC.1, 3 http://www.illustrativemathematics.org/illustrations/122 Musical Preferences: S-IC.1, S-ID.5 http://www.illustrativemathematics.org/illustrations/123 SAT Score: S.ID.4 http://www.illustrativemathematics.org/illustrations/216 Do You Fit In This Car?: S.ID.4 http://www.illustrativemathematics.org/illustrations/1020 Should We Send Out a Certificate?: S.ID.4 http://www.illustrativemathematics.org/illustrations/1218	Engage students in a discussion or activity to clearly distinguish between categorical and numerical variables by providing multiple examples of each type. Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's sport preferences) or measurement on the vertical (students' grade in a test). One tool for developing statistical models is the use of simulations. This allows the students to visualize the model and apply their understanding of the statistical process. Provide students the opportunities to distinguish between a population parameter which is a constant, and a sample statistic which is a variable. Use teacher-guided comparison conversations to ensure that students are able to make connections. As the statistical process is being mastered by students, it is important for them to investigate questions such as "If a coin spun five times produces five tails in a row, could one conclude that the coin is biased toward tails?" Students will need to use all of the data analysis, statistics, and probability concepts covered to date to develop a deeper understanding of inferential reasoning. Have students critique published surveys before having them design their own surveys. Unlike	SBAC - http://www.smarterbalanced.org/

	in observational studies; in surveys, the sample selected from a population needs to be representative of the population. Taking a random sample is generally what is done to satisfy this requirement. Use a variety of devices as appropriate to carry out simulations: number cubes, cards, random digit tables, graphing calculators, computer programs.	

LANGUAGE GOALS

- Students will describe orally and in writing the purposes of and differences among sample surveys, experiments, and observational studies.
  *Example: "*Based on the survey of teenage high school students, more students are more/less likely to _____ than _____."

- Students will decide whether a specified model is consistent with results from a data simulation.
  Example: "A model stating that a spinning coin falls heads up with a probability of 0.5 is not consistent with a simulation result of 5 tails in a row."

- Students will explain orally and in writing how they use statistical and probability concept in their lives, using the following specific set of words: *distribution, mean, standard deviation, probability,* and *statistics.*
  Example: "Based on the distribution of test scores with a mean of _____ and a standard deviation of _____, a test score of _____ is (*adjective).*
- Students will explain orally and in writing that areas under the normal curve allow us to answer and model real life situations.

PERFORMANCE TASKS

Mathematics Assessment Projects (MARS Tasks)

- **Modeling Conditional Probabilities 1: Lucky Dip: S.MD.6** http://map.mathshell.org/materials/lessons.php?taskid=409&subpage=problem

NCTM Illuminations Lessons

- **Should We Send a Certificate?**: S.ID.4 http://www.illustrativemathematics.org/illustrations/1218
- **Exploration with Chance:** S.ID.6 http://illuminations.nctm.org/LessonDetail.aspx?id=L290

Illuminations
Fred's Fun Factory: S-MD.2, 5 and 7
http://www.illustrativemathematics.org/illustrations/1197
Miscellaneous Sources

- **The Normal Distribution: S.ID.4** http://www.wmich.edu/cpmp/1st/unitsamples/pdfs/C3U5_362-375.pdf
- **Applications of Probability:** http://www.schools.utah.gov/CURR/mathsec/Core/Secondary-II/II-4-S-MD-H-6-and-7.aspx

DIFFERENTIATION📖		
FRONT LOADING	ACCELERATION	INTERVENTION
Students should be encouraged to persevere when problem solving in this unit. Multiple solutions are common and should be recognized. Students can often make sense of complex contextual probabilities by considering a simpler analogous Probability situation (MP.1). As students work to identify events for which probabilities are to be determined and rules to apply, encourage students to verify and critique the thinking of their classmates (MP.3). Students have the opportunity to demonstrate proficiency with MP.6 by paying close attention to precise use of new vocabulary and writing complete sentences describing probabilities.	S.MD.7 Apply this standard with more complex probability models. You can implement the following activity: But mango is my favorite… http://www.illustrativemathematics.org/illustrations/1333 Often two sample groups are compared in clinical studies. Two key criteria are specified: are the data normally distributed and are the data paired? Unpaired (independent) normally distributed data: Student's unpaired two-sample t-test For example, the efficacy of a new drug A may be compared with an established drug B. The study has 220 patients in treatment Group A with sample mean $\bar{x}A$ and standard deviation SDA and 200 patients in treatment Group B with sample mean $\bar{x}B$ and standard deviation SDB; (Group A and Group B do not have to be equal). We need to calculate the difference between the two sample means and the standard error of this difference between the two means, from which we can calculate a confidence interval for the difference between them. For t-test to be valid, the standard deviations of both groups must be similar. This is often the case, even when the sample means are significantly different.	Review the difference between independent events and dependent events. Review the conversions of: • Ratios • Percentages • Decimals Teach students how to understand data in multiple forms: • Graphs • Charts • Table Review key vocabulary words from previous sections

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